9

WORK AND KINETIC ENERGY

Conceptual Questions

9.1. We must calculate the new kinetic energy and compare it to the original value. Originally, $K=\frac{1}{2}mv^2$. With a velocity of $3v$, $K'=\frac{1}{2}m(3v)^2=9\left(\frac{1}{2}mv^2\right)=9K$. The kinetic energy increases by a factor of 9.

9.2. We have

$$K_\text{A}=8K_\text{B}$$

$$\frac{1}{2}m_\text{A}v_\text{A}^2=8\left(\frac{1}{2}m_\text{B}v_\text{B}^2\right)$$

Since $m_A=\frac{1}{2}m_B$,

$$\frac{1}{2}\left(\frac{1}{2}m_\text{B}\right)v_\text{A}^2=8\left(\frac{1}{2}m_\text{B}v_\text{B}^2\right)$$

$$\Rightarrow\frac{v_\text{A}}{v_\text{B}}=4$$

9.3. The work done by the tension in the cable is negative because the load's displacement is downward and the force is upward. The angle between them is $180°$ so the cosine of the angle is negative. The work done by the gravitational force is positive because the force is down and the displacement is down. The angle between them is zero so the cosine is positive.

9.4. The work done by the tension in the rope is positive because the force and the displacement are in the same direction; the angle between them is $0°$ so the cosine is positive.

9.5. The kinetic energies are equal. Equal forces are applied over equal displacements so that the same work is done on each. Thus, the change in kinetic energy is the same. Because $K_\text{i}=0$, $\Delta K=K_\text{f}$. (The plastic will be moving 10 times faster, however.)

9.6. The work done by the pushing force is negative because the displacement is to the left while the force is to the right. The cosine of the angle between them is negative. Another way to know is that the particle is slowing; this means negative work is being done on it.

9.7. No work was done by gravity. $W_g = -m_g\Delta y$. Here, $\Delta y = 0$. Any work done during a downward part of the motion was undone during the upward parts.

9.8. The work is the same in both cases, since the work done against gravity is $-m_g\Delta y$, and Δy, the change in height, is the same in both cases.

9.9. The tension does no work on the ball because the force is at right angles to the displacement. (The displacement is in the same direction as the velocity.) The cosine of $90°$ is zero so no work is done.

9.10. Because the sprinter is getting faster the work done must be positive. That is, since the kinetic energy is increasing the net work done on the sprinter must be positive.

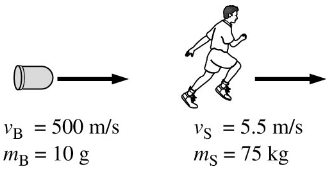
9.11. (a) We identify the equilibrium position $s_e = 10$ cm. At $s = 11$ cm, $\Delta s = s - s_e = 11\text{ cm} - 10\text{ cm} = 1$ cm, and $F_{sp} = F = -k\Delta s = -k(1\text{ cm})$. To get $F_{sp} = 3F$, we must have $3F = -k(\Delta s)'$, which means $(\Delta s)' = 3$ cm. So the spring must have length $10\text{ cm} + 3\text{ cm} = 13$ cm.
(b) Note that the direction of the force is reversed when the spring is compressed.
To get $F_{sp} = -2F$, we must have $-2F = -k(\Delta s)'$, which means $(\Delta s)' = -2$ cm. So the spring must have length $10\text{ cm} - 2\text{ cm} = 8$ cm.

9.12. Note that Carlos takes the place of the wall, and that the force on the spring is still 200 N. The spring still stretches 20 cm.

Exercises and Problems

Exercises

Section 9.2 Work and Kinetic Energy for a Single Particle
9.1. Model: We will use the particle model for the bullet (B) and the running student (S).
Visualize:



Solve: For the bullet,

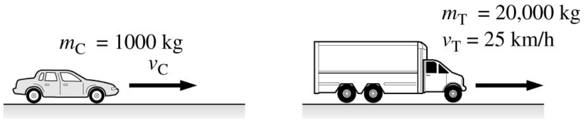
$$K_B = \frac{1}{2}m_B v_B^2 = \frac{1}{2}(0.010\text{ kg})(500\text{ m/s})^2 = 1250\text{ J}$$

For the running student,

$$K_S = \frac{1}{2}m_S v_S^2 = \frac{1}{2}(75\text{ kg})(5.5\text{ m/s})^2 = 206\text{ J}$$

Thus, the bullet has the larger kinetic energy.
Assess: Kinetic energy depends not only on mass but also on the square of the velocity. The above calculation shows this dependence. Although the mass of the bullet is 7500 times smaller than the mass of the student, its speed is more than 90 times larger.

9.2. Model: Model the compact car (C) and the truck (T) as particles.
Visualize:



Solve: For the kinetic energy of the compact car and the kinetic energy of the truck to be equal,

$$K_C = K_T \Rightarrow \frac{1}{2}m_C v_C^2 = \frac{1}{2}m_T v_T^2 \Rightarrow v_C = \sqrt{\frac{m_T}{m_C}}v_T = \sqrt{\frac{20{,}000 \text{ kg}}{1000 \text{ kg}}}(25 \text{ km/h}) = 112 \text{ km/h}$$

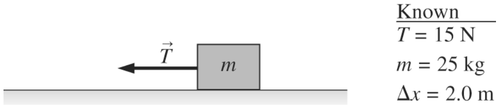
Assess: A smaller mass needs a greater velocity for its kinetic energy to be the same as that of a larger mass.

9.3. Model: Model the mother and the son as particles.
Visualize: $m_{\text{mother}} = 4\, m_{\text{son}}$.
Solve: The energy conservation equation $K_{\text{mother}} = K_{\text{son}}$ is

$$\frac{1}{2}m_{\text{mother}}v_{\text{mother}}^2 = \frac{1}{2}m_{\text{son}}v_{\text{son}}^2 \Rightarrow (4\, m_{\text{son}})v_{\text{mother}}^2 = m_{\text{son}}v_{\text{son}}^2 \Rightarrow \frac{v_{\text{son}}}{v_{\text{mother}}} = 2.0$$

Assess: The result $v_{\text{son}} = 2v_{\text{mother}}$, combined with the fact that $m_{\text{son}} = \frac{1}{4}m_{\text{mother}}$, is a consequence of the way kinetic energy is defined: It is directly proportional to the mass and to the square of the speed.

9.4. Model: The box is a particle with no air resistance.
Visualize: We are given $T = -15$ N, $m = 25$ kg, and $\Delta x = -2.0$ m.



Solve: (a) The work done by tension is

$$W = \int_{x_0}^{x_1} F_x dx = F_x \int_{x_0}^{x_1} dx = (-15 \text{ N})(-2.0 \text{ m}) = 30 \text{ J}$$

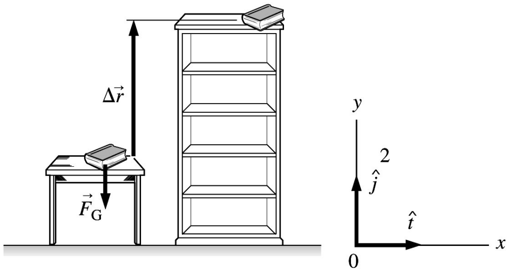
(b) The work done by gravity is 0 J because the force is perpendicular to the displacement.
Assess: A joule is a newton multiplied by a meter. We did not need the mass.

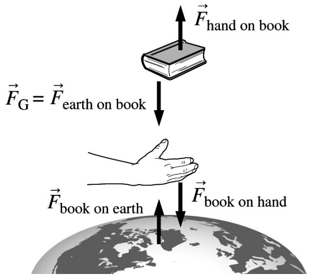
9.5. Model: The box is a particle with no air resistance.
Visualize: We are given $T = 15$ N, $m = 25$ kg, and $\Delta x = -0.35$ m.
Solve: (a) The work done by tension is

$$W = \int_{x_0}^{x_1} F_x dx = F_x \int_{x_0}^{x_1} dx = (15 \text{ N})(-0.35 \text{ m}) = 5.3 \text{ J}$$

(b) The work done by gravity is 0 J because the force is perpendicular to the displacement.
Assess: A joule is a newton multiplied by a meter. We did not need the mass.

9.6. Model: Work done by a force $\vec{F}$ on a particle is defined as $\vec{W} = \vec{F} \cdot \Delta\vec{r}$, where $\Delta\vec{r}$ is the particle's displacement.
Visualize:




$\vec{F}_{\text{hand on book}}$
$\vec{F}_G = \vec{F}_{\text{earth on book}}$
$\vec{F}_{\text{book on earth}}$
$\vec{F}_{\text{book on hand}}$

Solve: **(a)** The work done by gravity is

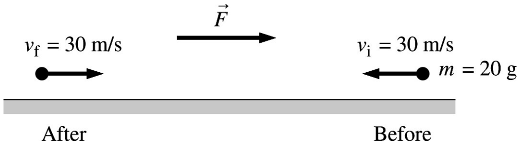
$$W_g = \vec{F}_G \cdot \Delta\vec{r} = (-mg\hat{j}) \cdot (2.25 - 0.75)\hat{j}\ \text{Nm} = -(2.0\ \text{kg})(9.8\ \text{m/s}^2)(1.50\ \text{m})\ \text{J} = -29\ \text{J}$$

(b) The work done by hand is $W_H = \vec{F}_{\text{hand on book}} \cdot \Delta\vec{r}$. As long as the book does not accelerate,

$$\vec{F}_{\text{hand on book}} = -\vec{F}_{\text{earth on book}} = -(-mg\hat{j}) = mg\hat{j}$$

$$W_H = (mg\hat{j}) \cdot (2.25 - 0.75)\hat{j}\ \text{Nm} = (2.0\ \text{kg})(9.8\ \text{m/s}^2)(1.50\ \text{m}) = 29\ \text{J}$$

9.7. Model: Use the work–kinetic energy theorem to find the net work done on the particle.
Visualize:



Solve: From the work–kinetic energy theorem,

$$W = \Delta K = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m(v_1^2 - v_0^2) = \frac{1}{2}(0.020\ \text{kg})\left[(30\ \text{m/s})^2 - (-30\ \text{m/s})\right]^2 = 0\ \text{J}$$

Assess: Negative work is done in slowing down the particle to rest, and an equal amount of positive work is done in bringing the particle to the original speed but in the opposite direction.

9.8. Model: Model the 2.0 kg object as a particle, and use the work–kinetic energy theorem.
Visualize: For each of the five intervals the kinetic energy-versus-time graph gives the initial and final kinetic energies. The mass of the object is 2.0 kg.
Solve: According to the work–kinetic energy theorem:

$$W = \Delta K$$

$$\text{Interval AB: } W = K_f - K_i = (0\ \text{J}) - (4\ \text{J}) = -4\ \text{J}$$

$$\text{Interval BC: } W = K_f - K_i = (0\ \text{J}) - (0\ \text{J}) = 0\ \text{J}$$

$$\text{Interval CD: } W = K_f - K_i = (2\ \text{J}) - (0\ \text{J}) = 2\ \text{J}$$

$$\text{Interval DE: } W = K_f - K_i = (4\ \text{J}) - (2\ \text{J}) = 2\ \text{J}$$

Assess: The total work done is zero because the final kinetic energy at time E is the same as it was at the beginning.

9.9. Model: Model the coin as a particle with no air resistance.
Visualize: We are given $v_{0y} = -4.0$ m/s and $y_0 = 35$ m, and $y_1 = 0$ m.
Solve: **(a)** Work done by gravity is the constant downward force multiplied by the downward distance.

$$W = (-mg) \cdot \Delta y = (0.0055\ \text{kg})(-9.8\ \text{m/s}^2)(-35\ \text{m}) = 1.9\ \text{J}$$

(b) The speed of the coin just as it hits the water can be found from kinematic equations.

$$v_\text{f}^2 = v_\text{i}^2 + 2a\Delta y \Rightarrow v_\text{f} = \sqrt{(-4.0\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)(-35\text{ m})} = 26.5\text{ m/s}$$

We report this as 26 m/s.
Assess: We use part **(b)** to check part **(a)**: According to the work–kinetic energy theorem

$$W = \Delta K = \tfrac{1}{2}m(v_\text{f}^2 - v_\text{i}^2) = \tfrac{1}{2}(0.0055\text{ kg})((-26.5\text{ m/s})^2 - (-4.0\text{ m/s})^2) = 1.9\text{ J}$$

9.10. Model: Model the girder as a particle with no air resistance.
Visualize: We are given $v_{0y} = -4.0$ m/s and $y_0 = 35$ m, and $y_1 = 0$ m.
Solve: (a) The gravitational force exerted by the earth is $F_\text{G} = mg$. The work done is then

$$W = \int_{y_0}^{y_1} F_y dy = F_y \Delta y = -mg\Delta y = -(750\text{ kg})(9.8\text{ m/s}^2)(3.5\text{ m}) = -25725\text{ J}$$

We report this as –26 kJ.
(b) The net (total) work done on the girder is its change of kinetic energy.

$$\Delta K = \tfrac{1}{2}m(v_\text{f}^2 - v_\text{i}^2) = \tfrac{1}{2}(750\text{ kg})((0.75\text{ m/s})^2 - (0.25\text{ m/s})^2) = 187.5\text{ J}$$

The work done by tension is the total work minus the work done by gravity.

$$W_\text{tension} = W_\text{total} - W_\text{grav} = 187.5\text{ J } - (-25725\text{ J}) = 25900\text{ J}$$

We report this as 26 kJ.
Assess: The magnitudes of the answers are the same because we rounded both to two significant figures.

Section 9.3 Calculating the Work Done

9.11. Solve: (a) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = (4)(-2) + (-2)(-3) = -2.$

(b) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = (-4)(2) + (2)(4) = 0.$

9.12. Solve: (a) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = (3)(2) + (4)(-6) = -18.$

(b) $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = (3)(6) + (-2)(4) = 10.$

9.13. Solve: (a) The length of $\vec{A}$ is $|\vec{A}| = A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5.$ The length of $\vec{B}$ is $B = \sqrt{(2)^2 + (-6)^2} = \sqrt{40} = 2\sqrt{10}.$ Using the answer $\vec{A}\cdot\vec{B} = -18$ from Exercise 9.12a,

$$\vec{A}\cdot\vec{B} = AB\cos\alpha$$
$$-18 = (5)(2\sqrt{10})\cos\alpha$$
$$\alpha = \cos^{-1}(-18/\sqrt{40}) = 125°$$

(b) The length of $\vec{A}$ is $|\vec{A}| = A = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{(3)^2 + (-2)^2} = \sqrt{13}.$ The length of $\vec{B}$ is $B = \sqrt{(6)^2 + (4)^2} = \sqrt{52} = 2\sqrt{13}.$

Using the answer $\vec{A}\cdot\vec{B} = 10$ from Exercise 9.12b,

$$\vec{A}\cdot\vec{B} = AB\cos\alpha$$
$$10 = (\sqrt{13})(2\sqrt{13})\cos\alpha$$
$$\alpha = \cos^{-1}(10/26) = 67°$$

9.14. Solve: (a) $\vec{A}\cdot\vec{B} = AB\cos\alpha = (5)(3)\cos 40° = 11.$

(b) $\vec{C}\cdot\vec{D} = CD\cos\alpha = (2)(3)\cos 140° = -4.6.$

(c) $\vec{E}\cdot\vec{F} = EF\cos\alpha = (3)(4)\cos 90° = 0.$

9.15. Solve: (a) $\vec{A}\cdot\vec{B} = AB\cos\alpha = (2)(4)\cos 110° = -2.7.$

(b) $\vec{C}\cdot\vec{D} = CD\cos\alpha = (5)(4)\cos 180° = -20.$

(c) $\vec{E}\cdot\vec{F} = EF\cos\alpha = (4)(3)\cos 30° = 10.$

9.16. Model: Assume the speed of the compressor is the same at the top and bottom of the incline.
Visualize: We are given $m = 25$ kg.
Solve:

$$W = \vec{F}\cdot\Delta\vec{r} = (25)(-9.8\hat{j})\cdot(7.0\hat{i} + 1.6\hat{j})\ \text{N m} = \left((25)(-9.8)(7.0)\overbrace{\hat{j}\cdot\hat{i}}^{=0} + (25)(-9.8)(1.6)\hat{j}\cdot\hat{j} \right)\text{J} = -390\ \text{J}.$$

Assess: We can neglect the horizontal displacement and the vertical force of gravity can be multiplied by the vertical displacement.

9.17. Model: Model the bug as a particle whose increase in speed is due entirely to the wind.
Visualize: We are given $m = 45$ g.
Solve: (a)

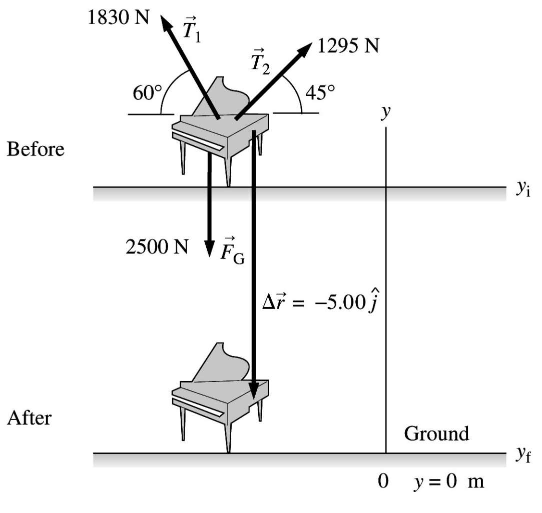
$$\vec{W} = \vec{F}\cdot\Delta\vec{r} = (4.0\hat{i} - 6.0\hat{j})\times 10^{-2}\ \text{N}\cdot(2.0\hat{i} - 2.0\hat{j})\ \text{m} = (0.080 + 0.12)\ \text{J} = 0.20\ \text{J}$$

(b) Recall that $v_\text{i} = 0$ m/s.

$$W = \Delta K = \tfrac{1}{2}m(v_\text{f}^2 - v_\text{i}^2) = \tfrac{1}{2}mv_\text{f}^2 \Rightarrow v_\text{f} = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(0.20\ \text{J})}{0.045\ \text{kg}}} = 3.0\ \text{m/s}$$

Assess: This seems like a good speed for a bug.

9.18. Model: Model the piano as a particle and use $W = \vec{F}\cdot\Delta\vec{r}$, where W is the work done by the force $\vec{F}$ through the displacement $\Delta\vec{r}$.
Visualize:



Solve: For the force $\vec{F}_G$:

$$W = \vec{F}\cdot\Delta\vec{r} = \vec{F}_G\cdot\Delta\vec{r} = (F_g)\cdot(\Delta r)\cos(0°) = (255\text{ kg})(9.81\text{ m/s}^2)(5.00\text{ m})(1.00) = 1.25\times10^4\text{ J}$$

For the tension $\vec{T}_1$:

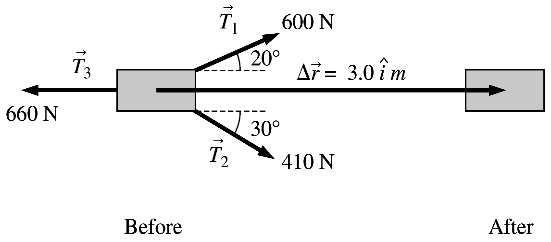
$$W = \vec{T}_1\cdot\Delta\vec{r} = (T_1)(\Delta r)\cos(150°) = (1830\text{ N})(5.00\text{ m})(-0.8660) = -7.92\times10^3\text{ J}$$

For the tension $\vec{T}_2$:

$$W = \vec{T}_2\cdot\Delta\vec{r} = (T_2)(\Delta r)\cos(135°) = (1295\text{ N})(5.00\text{ m})(-0.7071) = -4.58\times10^3\text{ J}$$

Assess: Note that the displacement $\Delta\vec{r}$ in all the above cases is directed downward along $-\hat{j}$.

9.19. Model: Model the crate as a particle and use $W = \vec{F}\cdot\Delta\vec{r}$, where W is the work done by a force $\vec{F}$ on a particle and $\Delta\vec{r}$ is the particle's displacement.
Visualize:



Solve: For the tension $\vec{T}_1$:

$$W = \vec{T}_1\cdot\Delta\vec{r} = (T_1)(\Delta r)\cos(20°) = (600\text{ N})(3.0\text{ m})(0.9397) = 1.7\text{ kJ}$$

For the tension $\vec{T}_2$:

$$W = \vec{T}_2\cdot\Delta\vec{r} = (T_2)(\Delta r)\cos(30°) = (410\text{ N})(3.0\text{ m})(0.866) = 1.1\text{ kJ}$$

For the force $\vec{T}_3$:

$$W = \vec{T}_3\cdot\Delta\vec{r} = (T_3)(\Delta r)\cos(180°) = (660\text{ N})(3.0\text{ m})(-1.0) = -2.0\text{ kJ}$$

Assess: Negative work done by the force of kinetic friction $\vec{T}_3$ means that 1.95 kJ of energy has been transferred *out* of the crate.

9.20. Model: Use the definition of work.
Visualize: Please refer to Figure EX9.20.
Solve: Work is defined as the area under the force-versus-position graph:

$$W = \int_{s_i}^{s_f} F_s ds = \text{area under the force curve}$$

$$\text{Interval } 0-1\text{ m: } W = (4\text{ N})(1\text{ m} - 0\text{ m}) = 4\text{ J}$$
$$\text{Interval } 1-2\text{ m: } W = (4\text{ N})(0.5\text{ m}) + (-4\text{ N})(0.5\text{ m}) = 0\text{ J}$$
$$\text{Interval } 2-3\text{ m: } W = \tfrac{1}{2}(-4.0\text{ N})(1\text{ m}) = -2\text{ J}$$

9.21. Model: Use the work–kinetic energy theorem to find velocities.
Solve: The work–kinetic energy theorem is

$$\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = W = \int_{x_i}^{x_f} F_x dx = \text{ area under the force curve from } x_i \text{ to } x_f$$

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}(0.500\text{ kg})(2.0\text{ m/s})^2 = \tfrac{1}{2}mv_f^2 - 1.0\text{ J} = \int_{x_i}^{x_f} F_x dx$$

$$\text{At } x = 1\text{ m: } \tfrac{1}{2}mv_f^2 - 1.0\text{ J} = 15\text{ N}\cdot\text{m} \;\Rightarrow\; v_f = 8.0\text{ m/s}$$

$$\text{At } x = 2\text{ m: } \tfrac{1}{2}mv_f^2 - 1.0\text{ J} = (15 + 11.25)\text{ N}\cdot\text{m} \;\Rightarrow\; v_f = 10\text{ m/s}$$

$$\text{At } x = 3\text{ m: } \tfrac{1}{2}mv_f^2 - 1.0\text{ J} = (15 + 11.25 + 3.75)\text{ N}\cdot\text{m} \;\Rightarrow\; v_f = 11\text{ m/s}$$

Assess: The speed is increasing as more work is done on it, but not linearly.

9.22. Model: Use the work–kinetic energy theorem.
Visualize: Please refer to Figure EX9.22.
Solve: The work–kinetic energy theorem is

$$\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \int_{0\text{ m}}^{x_f} F_x\, dx = 10x - \tfrac{5}{2}x^2$$

$$v_f = \sqrt{\frac{20x - 5x^2}{2.0\text{ kg}} + 4.0\text{ m/s}}$$

$$\text{At } x = 2\text{ m: } \Rightarrow\; v_f = 5\text{ m/s}$$

$$\text{At } x = 4\text{ m: } \Rightarrow\; v_f = 4\text{ m/s}$$

9.23. Visualize: We have a case with a non-constant force, so we must integrate.
Solve:

$$W = \int_{x_i}^{x_f} F_x dx = \int_0^d qx^2 dx = q\left[\tfrac{1}{3}x^3\right]_0^d = \tfrac{q}{3}d^3$$

9.24. Model: Assume the given force is the only force.
Visualize: We are given $m = 150$ g and $v_i = 2.00$ m/s.
Solve: Solve this in two steps; first find the work done by integrating the variable force.

$$W = \int_{x_i}^{x_f} F_x dx = \int_0^{3.14} (0.250\text{ N})\sin\left(\frac{x}{2.00\text{ m}}\right)dx = (0.250\text{ N})\left[(-2.00\text{ m})\cos\left(\frac{x}{2.00\text{ m}}\right)\right]_0^{3.14}$$
$$= (0.250\text{ N})(-2.00\text{ m})\left[0 - 1\right] = 0.500\text{ N}$$

Now use the work–kinetic energy theorem.

$$W = \Delta K = \tfrac{1}{2}m(v_f^2 - v_i^2) = \tfrac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(0.500\text{ J})}{0.150\text{ kg}} + (2.00\text{ m/s})^2} = 3.27\text{ m/s}$$

Assess: This seems like a reasonable speed.

Section 9.4 Restoring Forces and the Work Done by a Spring

9.25. Model: Assume the spring and athlete are stationary.
Visualize: The athlete must push as hard as the compressed spring pushes the other way.
Solve:

$$\sum F = F_{\text{sp}} - F_{\text{G}} = -k(\Delta y) - mg = 0 \Rightarrow k = \frac{-mg}{\Delta y} = \frac{-(2.2\text{ kg})(9.8\text{ m/s}^2)}{(0.29\text{ m} - 0.35\text{ m})} = 360\text{ N/m}$$

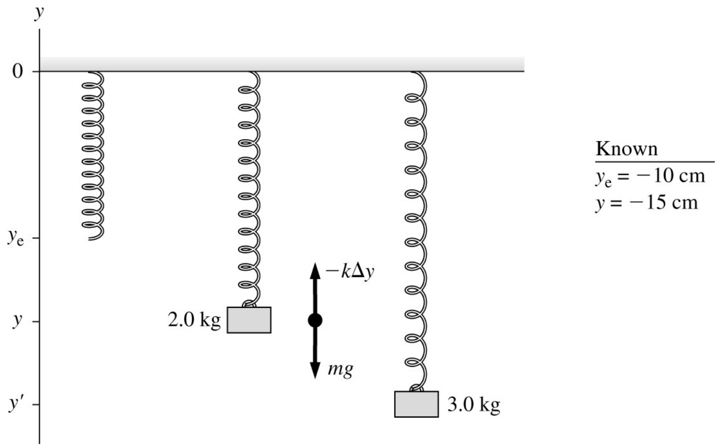
Assess: This seems like a reasonable speed.

9.26. Model: Assume the book is stationary so the net force on it is zero.
Visualize: The spring pushes up as much as gravity pulls down.
Solve:

$$F_{\text{athl}} = -F_{\text{sp}} = k(\Delta x) = (750\text{ N/m})(0.050\text{ m}) = 38\text{ N}$$

Assess: This seems like a reasonable speed.

9.27 Model: Assume an ideal spring that obeys Hooke's law.
Visualize:



Solve: (a) The spring force on the 2.0 kg mass is $F_{\text{sp}} = -k\Delta y$. Notice that Δy is negative, so F_{sp} is positive. This force is equal to *mg*, because the 2.0 kg mass is at rest. We have $-k\Delta y = mg$. Solving for *k*:

$$k = -(mg/\Delta y) = -(2.0\text{ kg})(9.8\text{ m/s}^2)/(-0.15\text{ m} - (-0.10\text{ m})) = 392\text{ N/m}$$

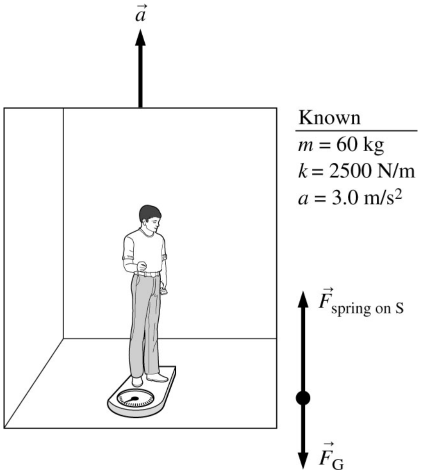
The spring constant is 3.9×10^2 N/m.
(b) Again using $-k\Delta y = mg$:

$$\Delta y = -mg/k = -(3.0\text{ kg})(9.8\text{ m/s}^2)/(392\text{ N/m})$$

$$y' - y_{\text{e}} = -0.075\text{ m} \Rightarrow y' = y_{\text{e}} - 0.075\text{ m} = -0.10\text{ m} - 0.075\text{ m} = -0.175\text{ m} = -17.5\text{ cm}$$

The length of the spring is 17.5 cm when a mass of 3.0 kg is attached to the spring. The *position* of the end of the spring is negative because it is below the origin, but length must be a positive number.

9.28. Model: Model the student (S) as a particle and the spring as obeying Hooke's law.
Visualize:



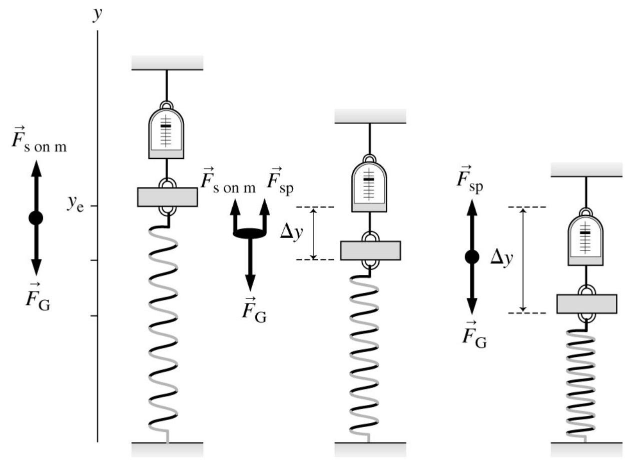
Solve: According to Newton's second law the force on the student is

$$\Sigma(F_{\text{on S}})_y = F_{\text{spring on S}} - F_{\text{G}} = ma_y$$

$$\Rightarrow F_{\text{spring on S}} = F_{\text{G}} + ma_y = mg + ma_y = (60\text{ kg})(9.8\text{ m/s}^2 + 3.0\text{ m/s}^2) = 768\text{ N}$$

Since $F_{\text{spring on S}} = F_{\text{S on spring}} = k\Delta y$, $k\Delta y = 768$ N. This means $\Delta y = (768\text{ N})/(2500\text{ N/m}) = 0.31$ m.

9.29. Model: Assume that the spring is ideal and obeys Hooke's law. We also model the 5.0 kg mass as a particle.
Visualize: We will use the subscript s for the scale and sp for the spring.



(a) **(b)** **(c)**

Solve: **(a)** The scale reads the upward force $F_{\text{s on m}}$ that it applies to the mass. Newton's second law gives

$$\Sigma(F_{\text{on m}})_y = F_{\text{s on m}} - F_{\text{G}} = 0 \Rightarrow F_{\text{s on m}} = F_{\text{G}} = mg = (5.0\text{ kg})(9.8\text{ m/s}^2) = 49\text{ N}$$

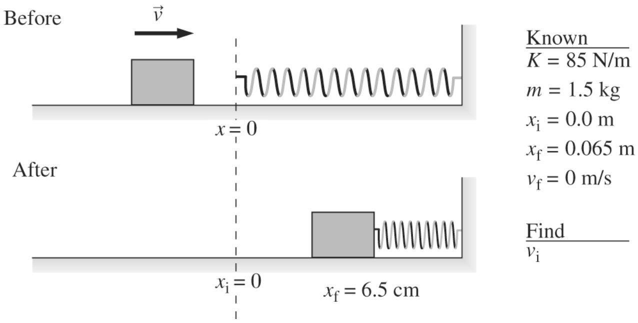
(b) In this case, the force is

$$\Sigma(F_{\text{on m}})_y = F_{\text{s on m}} + F_{\text{sp}} - F_{\text{G}} = 0 \Rightarrow 20\text{ N} + k\Delta y - mg = 0$$
$$\Rightarrow k = (mg - 20\text{ N})/\Delta y = (49\text{ N} - 20\text{ N})/0.02\text{ m} = 1450\text{ N/m}$$

The spring constant for the lower spring is 1.45×10^3 N/m.
(c) In this case, the force is

$$\Sigma(F_{\text{on m}})_y = F_{\text{sp}} - F_{\text{G}} = 0 \Rightarrow k\Delta y - mg = 0$$
$$\Rightarrow \Delta y = mg/k = (49\text{ N})/(1450\text{ N/m}) = 0.0338\text{ m} = 3.4\text{ cm}$$

9.30. Model: Assume the box is a particle.



Visualize: Use the work–kinetic energy theorem. The work comes from integrating a variable force.
Solve:

$$\int_0^{x_f} -kx\,dx = W = \Delta K = \tfrac{1}{2}m(v_f^2 - v_i^2)$$
$$-k\left[\tfrac{1}{2}x^2\right]_0^{0.065} = \tfrac{1}{2}m(0 - v_i^2)$$

Cancel 1/2 and a negative and solve for v_i.

$$v_i = \sqrt{\frac{k}{m}(0.065\text{ m})^2} = \sqrt{\frac{(85\text{ N/m})}{1.5\text{ kg}}(0.065\text{ m})^2} = 0.49\text{ m/s}$$

Assess: This seems like a reasonable speed.

Section 9.5 Dissipative Forces and Thermal Energy

9.31. Visualize: One mole of helium atoms in the gas phase contains $N_A = 6.02\times10^{23}$ atoms.

Solve: If each atom moves with the same speed v, the microscopic total kinetic energy will be

$$K_{\text{micro}} = N_A\left(\frac{1}{2}mv^2\right) = 3700\text{ J} \quad\Rightarrow\quad v = \sqrt{\frac{2K_{\text{micro}}}{mN_A}} = \sqrt{\frac{2(3700\text{ J})}{(6.68\times10^{-27}\text{ kg})(6.02\times10^{23})}} = 1360\text{ m/s}$$

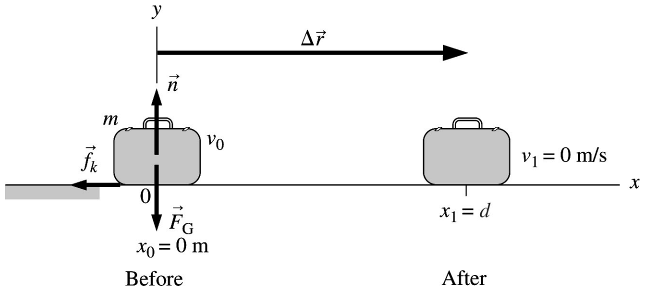
9.32. Model: The sliding is a dissipative force which transforms the kinetic energy to thermal energy. Assume the player ends up with zero speed.
Visualize: We are given $m = 55$ kg and $\Delta E_{\text{th}} = 950$ J. We seek v_i

Solve:
Cancel 1/2 and a negative and solve for v_i.

$$\Delta K = -\Delta E_\text{th} \Rightarrow \tfrac{1}{2}m(v_\text{f}^2 - v_\text{i}^2) = -950\ \text{J} \Rightarrow v_\text{i} = \sqrt{\frac{2(950\ \text{J})}{55\ \text{kg}}} = 5.9\ \text{m/s}$$

Assess: This seems like a reasonable speed for a softball player sliding into base.

9.33. Model: Model the suitcase as a particle, use the model of kinetic friction, and use the work–kinetic energy theorem.
Visualize:



The net force on the suitcase is $\vec{F}_\text{net} = \vec{f}_\text{k}$.
Solve: The work–kinetic energy theorem gives

$$W_\text{net} = \Delta K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 \quad \Rightarrow \quad \vec{F}_\text{net}\cdot\Delta\vec{r} = \vec{f}_\text{k}\cdot\Delta\vec{r} = 0\ \text{J} - \tfrac{1}{2}mv_0^2$$

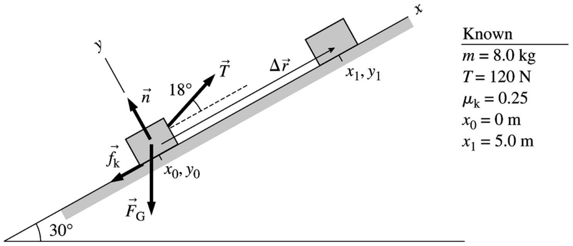
$$(f_\text{k})d\cos(180°) = -\tfrac{1}{2}mv_0^2 - \mu_\text{k}mgd = -\tfrac{1}{2}mv_0^2 \quad \Rightarrow \quad \mu_\text{k} = \frac{v_0^2}{2gd}$$

Inserting the given quantities into the expression for the coefficient of kinetic friction gives

$$\mu_\text{k} = \frac{v_0^2}{2gd} = \frac{(1.2\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)(2.0\ \text{m})} = 0.037$$

Assess: Friction transforms kinetic energy of the suitcase into thermal energy. In response, the suitcase slows down and comes to rest. Notice that the coefficient of friction does not depend on the mass of the object, which is reasonable.

9.34. Model: Use the particle model, the definition of work $W = \vec{F}\cdot\Delta\vec{s}$, and the model of kinetic friction.
Visualize: We place the coordinate frame on the incline so that its x-axis is along the incline.



Solve: (a)

$$W_\text{T} = \vec{T}\cdot\Delta\vec{r} = T\Delta x\cos(18°) = (120\ \text{N})(5.0\ \text{m})\cos(18°) = 0.57\ \text{kJ}$$

$$W_\text{g} = \vec{F}_\text{G}\cdot\Delta\vec{r} = mg\Delta x\cos(120°) = (8.0\ \text{kg})(9.8\ \text{m/s}^2)(5.0\ \text{m})\cos(120°) = -0.20\ \text{kJ}$$

$$W_\text{n} = \vec{n}\cdot\Delta\vec{r} = n\Delta x\cos(90°) = 0.0\ \text{J}$$

(b) The amount of energy transformed into thermal energy is $\Delta E_{\text{th}} = f_k \Delta x = \mu_k n \Delta x$.

To find n, we write Newton's second law as follows:

$$\sum F_y = n - F_G\cos(30°) + T\sin(18°) = 0\text{ N} \quad \Rightarrow \quad n = F_G\cos(30°) - T\sin(18°)$$

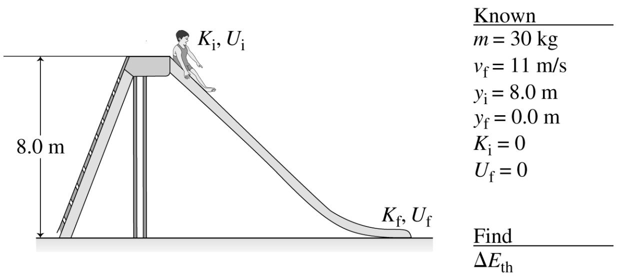
$$n = mg\cos(30°) - T\sin(18°) = (8.0\text{ kg})(9.8\text{ m/s}^2)\cos(30°) - (120\text{ N})\sin(18°) = 30.814\text{ N}$$

Thus, $\Delta E_{\text{th}} = (0.25)(30.814\text{ N})(5.0\text{ m}) = 39\text{ J}$.

Assess: Any force that acts perpendicular to the displacement does no work.

9.35. Model: Justin is a particle.

Visualize:



Solve:

$$K_i + U_i + W_{\text{ext}} = K_f + U_f + \Delta E_{\text{th}}$$

$$\Rightarrow \Delta E_{\text{th}} = U_i - K_f = mgy_i - \tfrac{1}{2}mv_f^2 = (30\text{ kg})(9.8\text{ m/s}^2)(8.0\text{ m}) - \tfrac{1}{2}(30\text{ kg})(11\text{ m/s})^2 = 540\text{ J}$$

Assess: This is about 23% of the initial potential energy, which is reasonable.

Section 9.6 Power

9.36. Model: Model the elevator as a particle, and apply the conservation of energy.

Solve: The tension in the cable does work on the elevator to lift it. Because the cable is pulled by the motor, we say that the motor does the work of lifting the elevator.

(a) The energy conservation equation is $K_i + U_i + W_{\text{ext}} = K_f + U_f + \Delta E_{\text{th}}$. Using $K_i = 0$ J, $K_f = 0$ J, and $\Delta E_{\text{th}} = 0$ J gives

$$W_{\text{ext}} = (U_f - U_i) = mg(y_f - y_i) = (1000\text{ kg})(9.8\text{ m/s}^2)(100\text{ m}) = 9.80\times10^5\text{ J}$$

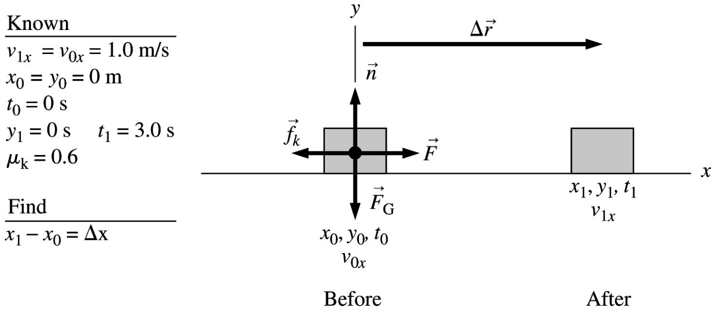
(b) The power required to give the elevator this much energy in a time of 50 s is

$$P = \frac{W_{\text{ext}}}{\Delta t} = \frac{9.80\times10^5\text{ J}}{50\text{ s}} = 1.96\times10^4\text{ W}$$

Assess: Since 1 horsepower (hp) is 746 W, the power of the motor is 26 hp. This is a reasonable amount of power to lift a mass of 1000 kg to a height of 100 m in 50 s.

9.37. Model: Model the steel block as a particle subject to the force of kinetic friction and use energy conservation.

Visualize:



Solve: (a) The work done on the block is $W_{\text{net}} = \vec{F}_{\text{net}} \cdot \Delta\vec{r}$ where $\Delta\vec{r}$ is the displacement. We will find the displacement using kinematic equations and the force using Newton's second law of motion. The displacement in the x-direction is

$$\Delta x = x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 = 0\text{ m} + (1.0\text{ m/s})(3.0\text{ s} - 0\text{ s}) + 0\text{ m} = 3.0\text{ m}$$

Thus $\Delta\vec{r} = 3.0\hat{i}$ m.
The equations for Newton's second law along the x and y components are

$$(F_{\text{net}})_y = n - F_{\text{G}} = 0\text{ N} \Rightarrow n = F_{\text{G}} = mg = (10\text{ kg})(9.8\text{ m/s}^2) = 98.0\text{ N}$$

$$(F_{\text{net}})_x = \vec{F} - \vec{f}_{\text{k}} = 0\text{ N} \Rightarrow F = f_{\text{k}} = \mu_{\text{k}} n = (0.6)(98.0\text{ N}) = 58.8\text{ N}$$

$$W_{\text{net}} = \vec{F}_{\text{net}} \cdot \Delta\vec{r} = F\Delta x\cos(0°) = (58.8\text{ N})(3.0\text{ m})(1) = 176\text{ J}$$

(b) The power required to do this much work in 3.0 s is

$$P = \frac{W}{t} = \frac{176\text{ J}}{3.0\text{ s}} = 59\text{ W}$$

9.38. Solve: The night light consumes more energy than the hair dryer. The calculations are

(a) $1.2\text{ kW} \times 10\text{ min} = 1.2 \times 10^3 \times 10 \times 60\text{ J} = 7.2 \times 10^5\text{ J}$

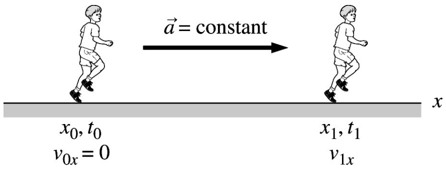
(b) $10\text{ W} \times 24\text{ hours} = 10 \times 24 \times 60 \times 60\text{ J} = 8.6 \times 10^5\text{ J}$

9.39. Solve: The power of the solar collector is the solar energy collected divided by time. The intensity of the solar energy striking the earth is the power divided by area. We have

$$P = \frac{\Delta E}{\Delta t} = \frac{150 \times 10^6\text{ J}}{3600\text{ s}} = 41{,}667\text{ W and intensity} = 1000\text{ W/m}^2$$

$$\text{Area of solar collector} = \frac{41{,}667\text{ W}}{1000\text{ W/m}^2} = 42\text{ m}^2$$

9.40. Model: Model the sprinter as a particle, and use the constant-acceleration kinematic equations and the definition of power in terms of velocity.
Visualize:



Solve: (a) We can find the acceleration from the kinematic equations and the horizontal force from Newton's second law. We have

$$x = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 \Rightarrow 50\text{ m} = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}a_x(7.0\text{ s} - 0\text{ s})^2 \Rightarrow a_x = 2.04\text{ m/s}^2$$

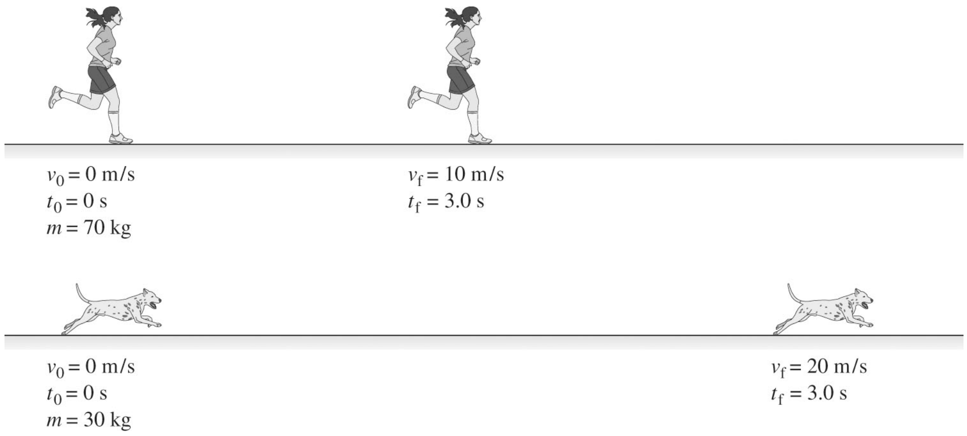
$$F_x = ma_x = (50\text{ kg})(2.04\text{ m/s}^2) = 10 \times 10^1\text{ N}$$

(b) We obtain the sprinter's power output by using $P = \vec{F} \cdot \vec{v}$, where $\vec{v}$ is the sprinter's velocity. At $t = 2.0$ s the power is

$$P = (F_x)[v_{0x} + a_x(t - t_0)] = (102\text{ N})[0\text{ m/s} + (2.04\text{ m/s}^2)(2.0\text{ s} - 0\text{ s})] = 0.42\text{ kW}$$

The power at $t = 4.0$ s is 0.83 kW, and at $t = 6.0$ s the power is 1.3 kW.

9.41. Visualize: See figure below.



Solve: Average power output is the change in energy of the system divided by the time interval. For the runner, the change in energy is just the change ΔK in the kinetic energy because the potential energy remains unchanged. Thus, $\Delta E = \Delta K = K_f - K_0 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}(70\text{ kg})(10\text{ m/s})^2 = 3500\text{ J}$. For the greyhound, the change in energy is $\Delta E = \Delta K = \frac{1}{2}(30\text{ kg})(20\text{ m/s})^2 = 6000\text{ J}$. Thus, the average power output of the runner is $\bar{P} = \Delta E/\Delta t = (3500\text{ J})/(3.0\text{ s}) = 1.2\text{ kW}$ and the average power output of the greyhound is $\bar{P} = (6000\text{ J})/(3.0\text{ s}) = 2.0\text{ kW}$.

9.42. Model: The heart provides the pressure to move blood through the body and therefore does work on the blood. We assume all the work goes into pushing the blood through the body.

Solve: (a) Using the hint, $W = PAd = PV = (1.3\times10^4\text{ N/m}^2)(6.0\times10^{-3}\text{ m}^3) = 78\text{ J}$ (in this equation, P represents pressure, not power).

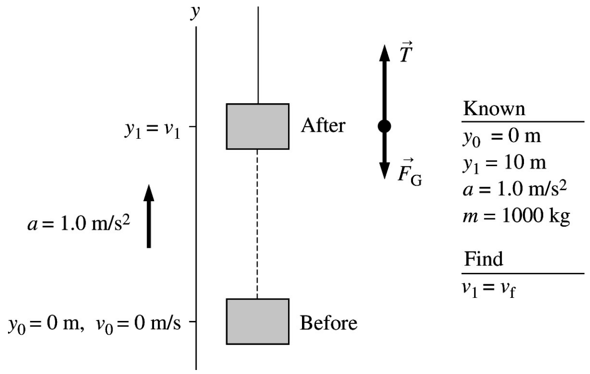
(b) Using *P* to represent power now, we can calculate the average power output of the heart as follows:

$$\bar{P} = \frac{W}{\Delta t} = \frac{78\text{ J}}{60\text{ s}} = 1.3\text{ W}$$

Problems

9.43. Model: Model the elevator as a particle.

Visualize:



Solve: (a) The work done by gravity on the elevator is

$$W_g = -\Delta U_g = mgy_0 - mgy_1 = -mg(y_1 - y_0) = -(1000\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = -9.8\times10^4\text{ J}$$

(b) The work done by the tension in the cable on the elevator is

$$W_T = T(\Delta y)\cos(0°) = T(y_1 - y_0) = T(10\text{ m})$$

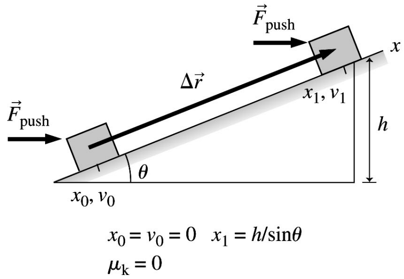
To find *T* we write Newton's second law for the elevator:

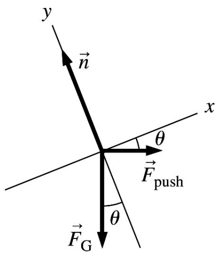
$$\sum F_y = T - F_G = ma_y \;\Rightarrow\; T = F_G + ma_y = m(g + a_y) = (1000\text{ kg})(9.8\text{ m/s}^2 + 1.0\text{ m/s}^2)$$
$$= 1.08\times10^4\text{ N} \;\Rightarrow\; W_T = (1.08\times10^4\text{ N})(10\text{ m}) = 1.1\times10^5\text{ J}$$

(c) The work–kinetic energy theorem is

$$W_{\text{net}} = W_g + W_T = \Delta K = K_f - K_i = K_f - \tfrac{1}{2}mv_0^2 \Rightarrow K_f = W_g + W_T + \tfrac{1}{2}mv_0^2$$
$$K_f = (-9.8\times10^4\text{ J}) + (1.08\times10^5\text{ J}) + \tfrac{1}{2}(1000\text{ kg})(0\text{ m/s})^2 = 1.0\times10^4\text{ J}$$

9.44. Model: Model the crate as a particle, and use the work–kinetic energy theorem.
Visualize:





Solve: (a) The work–kinetic energy theorem is $\Delta K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_1^2 = W_{\text{total}}$. Three forces act on the box, so $W_{\text{total}} = W_{\text{grav}} + W_n + W_{\text{push}}$. The normal force is perpendicular to the motion, so $W_n = 0$ J. The other two forces do the following amount of work:

$$W_{\text{push}} = \vec F_{\text{push}}\cdot\Delta\vec r = F_{\text{push}}x_1\cos\theta = F_{\text{push}}\left(\frac{h}{\sin\theta}\right)\cos\theta = F_{\text{push}}h\cot\theta$$

$$W_{\text{grav}} = \vec F_G\cdot\Delta\vec r = -mgx_1\sin\theta = -mg\left(\frac{h}{\sin\theta}\right)\sin\theta = -mgh$$

Thus, the speed at the top of the ramp is

$$v_1 = \sqrt{\frac{2W_{\text{total}}}{m}} = \sqrt{\frac{2(F_{\text{push}}h\cot\theta - mgh)}{m}}$$

(b) Insert the given quantities into the expression for the speed to find

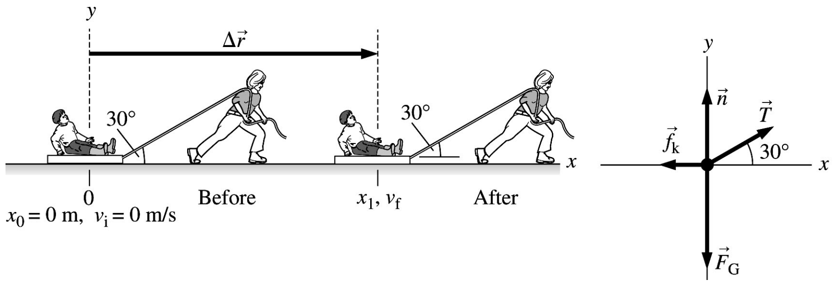
$$v_1 = \sqrt{\frac{2[(25\text{ N})(2.0\text{ m})\cot(20°) - (5.0\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m})]}{5.0\text{ kg}}} = 4.0\text{ m/s}$$

Assess: Note that $F_{\text{push}}\cos\theta > mg\sin\theta$ for the radical to remain positive. This means that the component of the pushing force up the slope must be greater than the component of gravity down the slope for the crate to move upward, which is the assumption with which we started. Furthermore, if we take the limit $h \to 0$, we get

$$\underset{\lim h\to0}{v_1} = \left.\sqrt{\frac{2\left[Fh\left(\frac{1}{h}\right) - mgh\right]}{m}}\,\right|_{h=0} = \sqrt{\frac{2F}{m}}$$

which is the expected result for pushing the crate along a horizontal frictionless surface.

9.45. Model: Model Paul and the mat as a particle, assume the mat to be massless, use the model of kinetic friction, and apply the work–kinetic energy theorem.
Visualize:



Known
$m = 10$ kg
$T = 30$ N
$x_0 = 0$ m, $x_1 = 3.0$ m
$v_i = 0$ m/s

Find
v_f

We define the x-axis along the floor and the y-axis perpendicular to the floor.
Solve: We first need to determine f_k. Newton's second law in the y-direction gives

$$n + T\sin(30°) = F_G = mg \quad\Rightarrow\quad n = mg - T\sin(30°) = (10\text{ kg})(9.8\text{ m/s}^2) - (30\text{ N})\sin(30°) = 83.0\text{ N}.$$

Using n and the model of kinetic friction gives $f_k = \mu_k n = (0.2)(83.0\text{ N}) = 16.60$ N. The net force on Paul and the mat is therefore $F_{net} = T\cos(30°) - f_k = (30\text{ N})\cos(30°) - 16.6\text{ N} = 9.4$ N. Thus,

$$W_{net} = F_{net}\Delta r = (9.4\text{ N})(3.0\text{ m}) = 28\text{ J}$$

The other forces $\vec{n}$ and $\vec{F}_G$ make an angle of 90° with $\Delta\vec{r}$ and do zero work. We can now use the work–kinetic energy theorem to find the final velocity as follows:

$$W_{net} = K_f - K_i = K_f - 0\text{ J} = \frac{1}{2}mv_f^2 \quad\Rightarrow\quad v_f = \sqrt{2W_{net}/m} = \sqrt{2(28\text{ J})/(10\text{ kg})} = 2.4\text{ m/s}$$

Assess: A speed of 2.4 m/s or 5.4 mph is reasonable for the present problem.

9.46. Model: The particle moves along the x-axis.
Visualize: When $x = 0$ then $v = 0$ and when $x = L$ then $v = v_0$.
Solve: Use the definition of work and the definition of acceleration. Move the dt under the dx and use $dx/dt = v$.

$$W = \int_{x_i}^{x_f} F_x dx = \int_{x_i}^{x_f} ma_x dx = \int_{x_i}^{x_f} m\left(\frac{dv}{dt}\right)dx = m\int_{x_i}^{x_f} dv\left(\frac{dx}{dt}\right) = m\int_{v_i}^{v_f} v\,dv = \frac{1}{2}m\left[\left(v_0\sin\left(\tfrac{\pi}{2L}x\right)\right)^2\right]_{x_i}^{x_f}$$

(a) Now plug in the values of v.

$$W = \frac{1}{2}m\left[\left(v_0\sin\left(\tfrac{\pi}{2L}x\right)\right)^2\right]_0^L = \frac{1}{2}mv_0^2\left[\left(\sin\left(\tfrac{\pi}{2L}L\right)\right)^2 - \left(\sin\left(\tfrac{\pi}{2L}0\right)\right)^2\right] = \frac{1}{2}mv_0^2[1-0] = \frac{1}{2}mv_0^2$$

(b) Now plug in the new values of v.

$$W = \frac{1}{2}m\left[\left(v_0\sin\left(\tfrac{\pi}{2L}x\right)\right)^2\right]_0^{2L} = \frac{1}{2}mv_0^2\left[\left(\sin\left(\tfrac{\pi}{2L}2L\right)\right)^2 - \left(\sin\left(\tfrac{\pi}{2L}0\right)\right)^2\right] = \frac{1}{2}mv_0^2[0-0] = 0$$

Assess: The work done between L and $2L$ is negative.

9.47. Model: The ball is a particle. Ignore air resistance.
Visualize: The ball starts with some kinetic energy which is completely transformed into potential energy which contains $\Delta y = h$.
Solve: If the ball starts with twice as much kinetic energy then it still have twice as much potential energy at the top, so it will reach twice as high, or $2h$.
Assess: Starting with twice the kinetic energy means four times the speed.

9.48. Model: Assume the pile driver is in free fall before it hits the pipe so all the potential energy is transformed into kinetic energy.

Visualize: Use the work–kinetic energy theorem on the pile driver. $U_f = K_i = 0$.

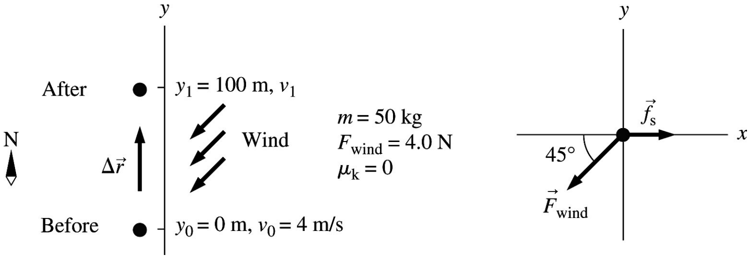
Solve: The pile driver has potential energy, which is transformed into kinetic energy as it falls.

$$U_i = K_f$$
$$\Delta K = K_f - K_i = W = F\Delta y_{\text{pipe}}$$
$$F = \frac{K_f}{\Delta y_{\text{pipe}}} = \frac{U_i}{\Delta y_{\text{pipe}}} = \frac{mgy_{\text{driver}}}{\Delta y_{\text{pipe}}} = \frac{(250 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m})}{0.35 \text{ m}} = 11 \text{ kN}$$

Assess: This is a lot of force, but that's what pile drivers are designed to do.

9.49. Model: Use the particle model for the ice skater, the friction model, and the work–kinetic energy theorem.

Visualize:



Solve: (a) The work–kinetic energy theorem gives

$$\Delta K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 = W_{\text{net}} = W_{\text{wind}}$$

There is no kinetic friction along her direction of motion. Static friction acts to prevent her skates from slipping sideways on the ice, but this force is perpendicular to the motion and does not contribute to a change in thermal energy. The angle between $\vec{F}_{\text{wind}}$ and $\Delta\vec{r}$ is $\theta = 135°$, so

$$W_{\text{wind}} = \vec{F}_{\text{wind}} \cdot \Delta\vec{r} = F_{\text{wind}}\Delta y\cos(135°) = (4.0 \text{ N})(100 \text{ m})\cos(135°) = -282.8 \text{ J}$$

Thus, her final speed is

$$v_1 = \sqrt{v_0^2 + \frac{2W_{\text{wind}}}{m}} = 2.2 \text{ m/s}$$

(b) If the skates don't slip, she has no acceleration in the x-direction and so $(F_{\text{net}})_x = 0$ N. That is:

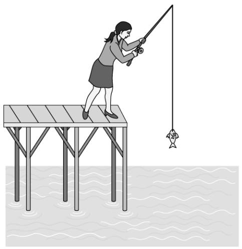
$$f_s - F_{\text{wind}}\cos(45°) = 0 \text{ N} \quad \Rightarrow \quad f_s = F_{\text{wind}}\cos(45°) = 2.83 \text{ N}$$

Now there is an upper limit to the static friction: $f_s \le (f_s)_{\max} = \mu_s mg$. To not slip requires

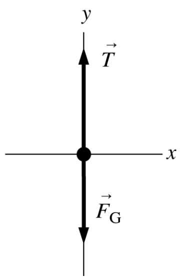
$$\mu_s \ge \frac{f_s}{mg} = \frac{2.83 \text{ N}}{(50 \text{ kg})(9.8 \text{ m/s}^2)} = 0.0058$$

Thus, the minimum value of μ_s is 0.0058.

9.50. Model: Assume the fish is lifted vertically and that the tension is constant.
Visualize:



Free-body diagram



Known
$T = 16$ N
$m = 1.5$ kg
$\Delta y = 2.0$ m

Find
v_f

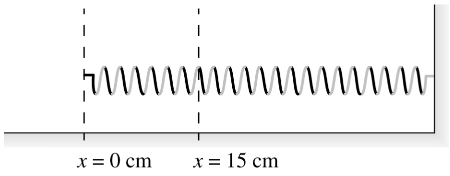
Solve: The pile driver has potential energy which is transformed into kinetic energy as it falls.

$$W_{\text{net}} = \Delta K \Rightarrow F_{\text{net}}\Delta y = \Delta K \Rightarrow (T - mg)\Delta y = \tfrac{1}{2}m(v_f^2 - v_i^2) = \tfrac{1}{2}mv_f^2$$

$$\Rightarrow v_f = \sqrt{\frac{2\Delta y(T - mg)}{m}} = \sqrt{\frac{2(2.0\text{ m})(16\text{ N} - (1.5\text{ kg})(9.8\text{ m/s}^2))}{1.5\text{ kg}}} = 1.9\text{ m/s}$$

Assess: These are reasonable numbers for a 3-lb fish.

9.51. Model: The spring is not ideal in part **(a)**.
Visualize: The work we have to do is the negative of the work done by the spring so we'll remove the negative signs from the force equation.



Known
$\Delta x = 15$ cm
$k = 250$ N/m
$q = 800$ N/m^3

Solve: (a)

$$W = \int_{x_0}^{x_1} F_x dx = \int_{x_0}^{x_1} (kx + qx^3)dx = \left[\tfrac{1}{2}kx^2 + \tfrac{1}{4}qx^4\right]_{x_0}^{x_1}$$

$$= \left[\tfrac{1}{2}(250\text{ N/m})x^2 + \tfrac{1}{4}(800\text{ N/m}^3)x^4\right]_0^{0.15} = \left[2.8125 + 0.10125\right] = 2.9\text{ J}$$

(b) We find the percent the second term is of the first: 0.10125 J /2.8125 J = 3.6%.
Assess: It would be important to make this correction for precise work.

9.52. Model: The particle is moving along the *x*-axis.
Visualize: The force is exponentially decreasing with *x*.
Solve:

$$W = \int_{x_0}^{x_1} F_x dx = \int_{x_0}^{x_1} F_0 e^{-x/L} dx = F_0\left[-Le^{-x/L}\right]_0^L = F_0 L\left[-e^{-L/L} + e^0\right] = F_0 L\left(1 - \frac{1}{e}\right)$$

Assess: The answer scales with *L*, as we would expect.

9.53. Model: Ignore forces due to any other objects, including Mars and Mercury.
Visualize: The work we have to do is the negative of the work done by the spring so we'll remove the negative signs from the force equation.
Solve: (a)

$$W = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} \left(-G\frac{m_1 m_2}{x^2} \right) dx = -G m_1 m_2 \left[-\frac{1}{x} \right]_{x_0}^{x_1} = G m_1 m_2 \left[-\frac{1}{x_1} + \frac{1}{x_2} \right] = G m_1 m_2 \left(\frac{x_2 - x_1}{x_1 x_2} \right)$$

(b) Look up $m_s = 1.99\times10^{30}\ \text{kg},\ m_c = 1.5\times10^{13}\ \text{kg},\ x_2 = 2.28\times10^{11}\ \text{m},\ x_1 = 5.79\times10^{9}\ \text{m}.$

$$W = \Delta K \Rightarrow G m_s m_c \left(\frac{x_2 - x_1}{x_1 x_2} \right) = \tfrac{1}{2} m_c (v_f^2 - v_i^2)$$

$$\Rightarrow v_f = \sqrt{2 G m_s \frac{x_2 - x_1}{x_1 x_2} + v_i^2}$$

$$= \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg}) \frac{(2.28\times10^{11}\ \text{m}) - (5.79\times10^{9}\ \text{m})}{(2.28\times10^{11}\ \text{m})(5.79\times10^{9}\ \text{m})} + (3.5\times10^{4}\ \text{m/s})^2}$$

$$= 2.1\times10^{5}\ \text{m/s}$$

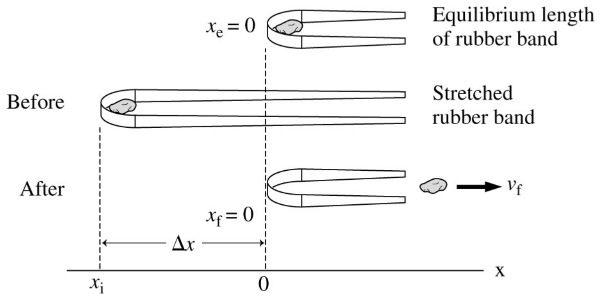
Assess: It would be important to make this correction for precise work.

9.54. Model: Ignore forces due to any other objects.
Visualize: The work we have to do is the negative of the work done by the spring so we'll remove the negative signs from the force equation.
Solve:

$$W = \int_{x_1}^{x_2} F_x dx = kqQd \int_{x_1}^{x_2} \left(\frac{1}{x^3} \right) dx = kqQd \left[-\frac{1}{x^2} \right]_{x_1}^{x_2} = kqQd \left[-\frac{1}{x_2^2} + \frac{1}{x_1^2} \right] = kqQd \left(\frac{x_2^2 - x_1^2}{x_1^2 x_2^2} \right)$$

Assess: It would be important to make this correction for precise work.

9.55. Model: Assume that the rubber band behaves similar to a spring. Also, model the rock as a particle.
Visualize:



Solve: From the graph, $|\Delta F_{sp}| = 20\ \text{N}$ for $|\Delta x| = 10\ \text{cm}$. Thus,

$$k = \frac{|\Delta F_{sp}|}{|\Delta x|} = \frac{20\ \text{N}}{0.10\ \text{m}} = 200\ \text{N/m} = 2.0\times10^{2}\ \text{N/m}$$

The conservation of energy equation $K_f + U_{sf} = K_i + U_{si}$ for the rock is

$$\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 \Rightarrow \frac{1}{2}mv_f^2 + \frac{1}{2}k(0\text{ m})^2 = \frac{1}{2}m(0\text{ m/s})^2 + \frac{1}{2}kx_i^2$$

$$v_f = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{200\text{ N/m}}{0.050\text{ kg}}}(0.30\text{ m}) = 19\text{ m/s}$$

Assess: Note that x_i is Δx, which is the displacement relative to the equilibrium position, and x_f is the equilibrium position of the rubber band, which is equal to zero.

9.56. Model: The two cheerleaders are particles.
Visualize: We are given $m_1 = 65$ kg, $\Delta y_1 = 5.5$ cm, $\Delta y_2 = 4.5$ cm. We seek m_2.
Solve: Use Hooke's law for each situation: $-k = F/\Delta s$.

$$-k = \frac{m_1 g}{\Delta y_1} = \frac{(m_1+m_2)g}{\Delta y_1 + \Delta y_2} \Rightarrow m_2 = m_1\left(\frac{\Delta y_1 + \Delta y_2}{\Delta y_1} - 1\right) = (65\text{ kg})\left(\frac{5.5\text{ cm} + 4.5\text{ cm}}{5.5\text{ cm}} - 1\right) = 53\text{ kg}$$

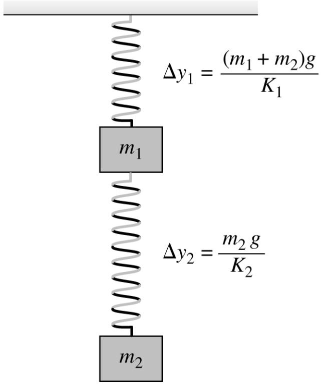
Assess: It makes sense to put the lighter cheerleader on top.

9.57. Model: The two springs are ideal.
Visualize: Since one spring is compressed 3.0 cm and the other stretched by 3.0 cm, the total amount of spring stretching/compressing is 6.0 cm.
Solve: Use Hooke's law: $k = -F/\Delta s$.

$$k = \frac{-F_{sp}}{\Delta x} = \frac{2.0\text{ N}}{0.060\text{ m}} = 33\text{ N/m}$$

Assess: This is a typical spring constant.

9.58. Model: The two masses are particles with no size. The springs are ideal.
Visualize: Both masses are stretching the top spring.

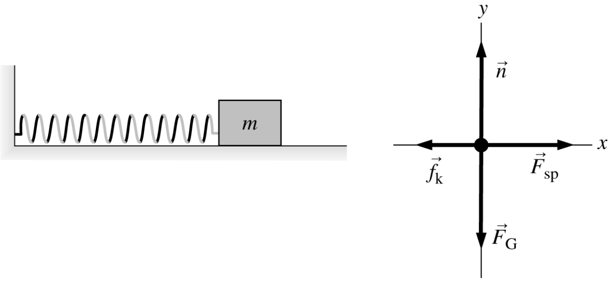

$\Delta y_1 = \frac{(m_1+m_2)g}{K_1}$
m_1
$\Delta y_2 = \frac{m_2 g}{K_2}$
m_2

Solve: The total length (how far m_2 is below the ceiling) is the original lengths plus the stretch of each spring.

$$L_1 + L_2 + \Delta y_1 + \Delta y_2 = L_1 + L_2 + \frac{(m_1+m_2)g}{k_1} + \frac{m_2 g}{k_2} = L_1 + L_2 + \left(\frac{m_1+m_2}{k_1} + \frac{m_2}{k_2}\right)g$$

Assess: The bottom mass stretches both springs.

9.59. Model: The box starts from rest.
Visualize: Use the work–kinetic energy theorem



Known
$k = 250$ N/m
$\Delta x = 0.12$ m
$m = 0.25$ kg
$\mu_k = 0.23$

Solve: First compute the total work done on the box during the launch.

$$W = \int_{x_0}^{x_1} (F_{sp} - f_k)dx = \int_{x_0}^{x_1} (kx - \mu_k n)dx = \left[\tfrac{1}{2}kx^2 - \mu_k mgx\right]_{x_0}^{x_1}$$

$$= \left[\tfrac{1}{2}(250 \text{ N/m})x^2 - (0.23)(0.25 \text{ kg})(9.8 \text{ m/s}^2)x\right]_0^{0.12} = 1.73 \text{ J}$$

Now use the work–kinetic energy theorem.

$$1.73 \text{ J} = \tfrac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2(1.73 \text{ J})}{0.25 \text{ kg}}} = 3.7 \text{ m/s}$$

Assess: The friction decreased the launch speed only a bit.

9.60. Model: The firefighter is a particle.
Visualize: The work done by the firefighter will equal the negative of the work the force of gravity does.
Solve: The power is the work done divided by the time it takes.

$$P = \frac{W}{\Delta t} = \frac{mg\Delta y}{\Delta t} = \frac{(90 \text{ kg} + 40 \text{ kg})(9.8 \text{ m/s}^2)(20 \text{ m})}{55 \text{ s}} = 460 \text{ W}$$

Assess: This power output is heavy exertion, but reasonable in the line of duty of a firefighter.

9.61. Model: Assume the water starts from rest.
Visualize: 80% of 50 MW is 40 MW.
Solve: Use the work–kinetic energy theorem.

$$P = \frac{W}{\Delta t} = \frac{\frac{1}{2}mv^2}{\Delta t} \Rightarrow \frac{m}{\Delta t} = \frac{2P}{v^2} = \frac{2(40 \text{ MW})}{(18 \text{ m/s})^2} = 2.5 \times 10^5 \text{ kg/s}$$

Assess: That seems like a lot of water per second.

9.64. Model: We will ignore rolling friction because it is much less than the drag force (and because we are not given the mass of the bicyclist + bicycle). Therefore, model the system as a particle with the given cross-sectional area and that is moving through the air at the given speed.
Solve: (a) From Equation. 6.15, we know that the drag force D has the magnitude

$$D = \tfrac{1}{2}C\rho Av^2$$

where $C = 0.90$, $A = 0.45 \text{ m}^2$, and $\rho = 1.2 \text{ kg/m}^3$ (the density of air). To overcome this force, the cyclist must generate the force $\vec{F} = -\vec{D}$, or a power $P = \vec{F} \cdot \vec{v} = Dv$, where the last equality follows because the drag force acts in parallel to the velocity. Thus, the power is

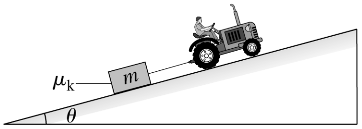
$$P = \tfrac{1}{2}C\rho Av^3 = \tfrac{1}{2}(0.90)(1.2 \text{ kg/m}^3)(0.45 \text{ m}^2)(7.3 \text{ m/s})^3 = 95 \text{ W}$$

(b) The metabolic power output P_M is $P_M = P/0.25 = 3.8\times10^2$ W.

(c) The number of food calories c burned riding for one hour is

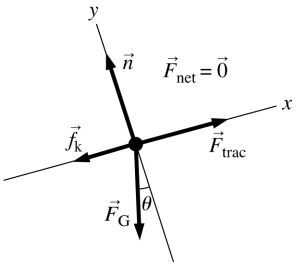
$$c = (378\text{ W})\left(\frac{1\text{ cal}}{4190\text{ J}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)(1\text{ h}) = 3.2\times10^2\text{ cal}$$

9.63. Model: Model the bale as a particle. Assume the tractor's power output all goes into moving the bale.
Visualize: Use tilted axes.



Known
$m = 150$ kg
$\mu_k = 0.45$
$\theta = 15°$

Find
P



Solve: We need to use Newton's second law in both directions.

$$\Sigma F_y = n - mg\cos\theta = 0 \Rightarrow n = mg\cos\theta$$

$$\Sigma F_x = F_{trac} - mg\sin\theta - f_k = 0 \Rightarrow F_{trac} = mg\sin\theta + \mu_k mg\cos\theta$$

For a constant force and constant velocity

$$P_{trac} = \frac{W_{trac}}{\Delta t} = \frac{\vec{F}_{trac}\cdot\Delta\vec{x}}{\Delta t} = \vec{F}_{trac}\cdot\vec{v} = mg(\sin\theta + \mu_k\cos\theta)v$$

$$= (150\text{ kg})(9.8\text{ m/s}^2)(\sin15° + 0.45\cos15°)(5.0\text{ km/h})\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 1.4\text{ kW}$$

Assess: 1.4 kW seems about right for a tractor.

9.64. Model: Use the model of static friction, kinematic equations, and the definition of power.
Solve: (a) The rated power of the Porsche is $217\text{ hp} = 161{,}882\text{ W}$ and the gravitational force on the car is $(1480\text{ kg})(9.8\text{ m/s}^2) = 14{,}504\text{ N}$. The amount of that force on the drive wheels is $(14{,}504)(2/3) = 9670\text{ N}$. Because the static friction of the tires on road pushes the car forward,

$$F_{max} = f_{s,max} = \mu_s n = \mu_s mg = (1.00)(9670\text{ N}) = ma_{max}$$

$$a_{max} = \frac{9670\text{ N}}{1480\text{ kg}} = 6.53\text{ m/s}^2$$

(b) Only 70% of the power generated by the motor is applied at the wheels.

$$P = Fv_{max} \quad\Rightarrow\quad v_{max} = \frac{P}{F} = \frac{(0.70)(161{,}882\text{ W})}{9670\text{ N}} = 11.7\text{ m/s}$$

(c) Using the kinematic equation, $v_{max} = v_0 + a_{max}(t_{min} - t_0)$ with $v_0 = 0$ m/s and $t_0 = 0$ s, we obtain

$$t_{min} = \frac{v_{max}}{a_{max}} = \frac{11.7\text{ m/s}}{6.53\text{ m/s}^2} = 1.79\text{ s}$$

Assess: An acceleration time of 1.79 s for the Porsche to reach a speed of ≈ 26 mph from rest is reasonable.

9.65. Model: The energy produced in the supernova explosion must all pass through an imaginary spherical surface whose radius is the distance to earth.

Visualize: Call the distance from the supernova to the earth d and the radius of the telescope r. We are given $E_{\text{tele}} = 9.1\times10^{-11}$ J, $E_{\text{super}} = (5.0\times10^{37}\text{ W})(10\text{ s}) = 5.0\times10^{38}$ J

Solve: Use ratios.

$$\frac{E_{\text{tele}}}{E_{\text{super}}} = \frac{A_{\text{tele}}}{A_{\text{super}}} \Rightarrow A_{\text{super}} = A_{\text{tele}}\frac{E_{\text{super}}}{E_{\text{tele}}} \Rightarrow 4\pi d^2 = \pi r^2\frac{E_{\text{super}}}{E_{\text{tele}}}$$

$$\Rightarrow d = \frac{r}{2}\sqrt{\frac{E_{\text{super}}}{E_{\text{tele}}}} = \frac{1.0\text{ m}}{2}\sqrt{\frac{(5.0\times10^{38}\text{ J})}{(9.1\times10^{-11}\text{ J})}} = 1.2\times10^{8}\text{ ly}$$

Assess: 120 million light years is very far indeed, but supernovae can be seen that far away because they are so bright.

9.66. Model: The sled is undergoing constant acceleration so the kinematic equations apply.

Visualize: We are given $m = 220\text{ kg},\quad \mu_{\text{k}} = 0.080.$

Solve: (a) We need to compute how far the sled travels during the acceleration phase starting from rest.

$$v_{\text{f}}^2 = 2a\Delta x \Rightarrow \Delta x = \frac{v_{\text{f}}^2}{2a} = \frac{\left((12\text{ km/h})\left(\frac{1\text{ h}}{3600\text{ s}}\right)\right)^2}{2(0.75\text{ m/s}^2)} = 7.407\text{ m}$$

We also find the time taken to reach cruising speed from rest.

$$\Delta x = \tfrac{1}{2}a(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2\Delta x}{a}} = \sqrt{\frac{2(7.407\text{ m})}{(0.75\text{ m/s}^2)}} = 4.44\text{ s}$$

Now find the force the dogs pull with during the acceleration phase.

$$\Sigma F_y = n - mg = 0 \Rightarrow n = mg$$

$$\Sigma F_x = F_{\text{dogs}} - f_{\text{k}} = ma \Rightarrow F_{\text{dogs}} = ma + \mu_{\text{k}}mg = m(a + \mu_{\text{k}}g)$$

$$= (220\text{ kg})(0.75\text{ m/s}^2 + 0.080(9.8\text{ m/s}^2)) = 337\text{ N}$$

Now put it all together to find the power output of the dogs.

$$P_{\text{dogs}} = \frac{W_{\text{dogs}}}{\Delta t} = \frac{F_{\text{dogs}}\Delta x}{\Delta t} = \frac{(337\text{ N})(7.407\text{ m})}{4.44\text{ s}} = 560\text{ W}$$

(b) At cruising speed the calculation is easier because the acceleration is easier.

$$\Sigma F_y = n - mg = 0 \Rightarrow n = mg$$

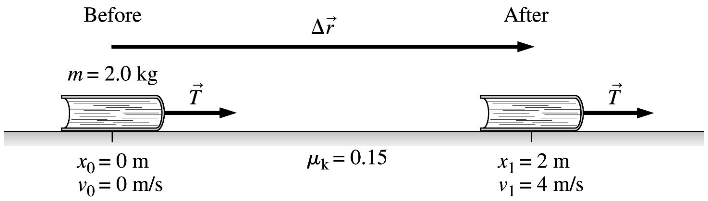
$$\Sigma F_x = F_{\text{dogs}} - f_{\text{k}} = 0 \Rightarrow F_{\text{dogs}} = \mu_{\text{k}}mg = (0.080)(220\text{ kg})(9.8\text{ m/s}^2) = 172.5\text{ N}$$

$$P_{\text{dogs}} = \frac{W_{\text{dogs}}}{\Delta t} = \frac{\vec{F}_{\text{dogs}}\cdot\Delta\vec{x}}{\Delta t} = \vec{F}_{\text{dogs}}\cdot\vec{v} = (172.5\text{ N})(12\text{ km/h})\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 570\text{ W}$$

Assess: It is a bit surprising that the power output of the dogs is greater when cruising at constant speed, but they must keep that high speed against friction.

9.67. Solve: (a) A student uses a string to pull her 2.0 kg physics book, starting from rest, across a 2.0-m-long lab bench. The coefficient of kinetic friction between the book and the lab bench is 0.15. If the book's final speed is 4.0 m/s, what is the tension in the string?

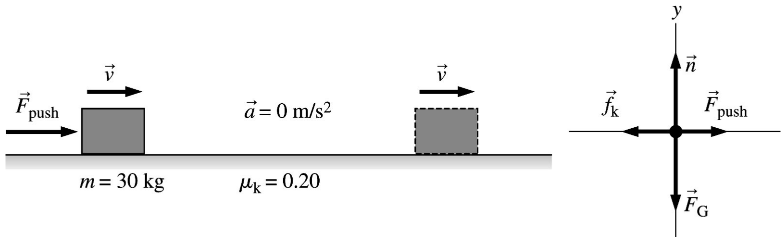
(b)



(c) The tension does external work W_{ext}. This work increases the book's kinetic energy and also causes an increase ΔE_{th} in the thermal energy of the book and the lab bench. Solving the equation gives $T = 11$ N.

9.68. **(a)** If you expend 75 W of power to push a 30 kg sled on a surface where the coefficient of kinetic friction between the sled and the surface is $\mu_k = 0.20$, what speed will you be able to maintain?

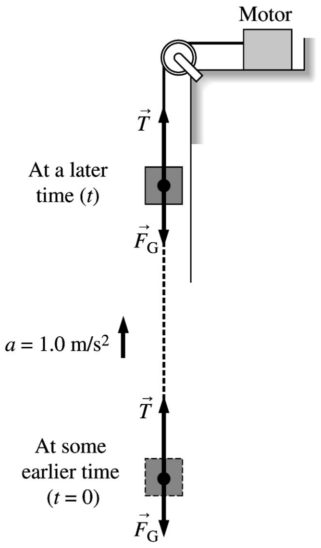
(b)



(c) $F_{\text{push}} = (0.20)(30\text{ kg})(9.8\text{ m/s}^2) = 58.8\text{ N} \;\Rightarrow\; 75\text{ W} = (58.8\text{ N})v \;\Rightarrow\; v = \dfrac{75\text{ W}}{58.8\text{ N}} = 1.3\text{ m/s}$

9.69. **(a)** A 1500 kg object is being accelerated upward at 1.0 m/s^2 by a rope. How much power must the motor supply at the instant when the velocity is 2.0 m/s?

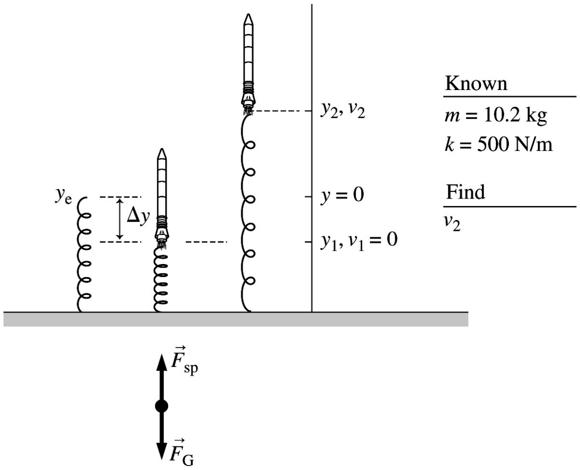
(b)



(c) $T = (1500\text{ kg})(9.8\text{ m/s}^2) + 1500\text{ kg}(1.0\text{ m/s}^2) = 16{,}200\text{ N} = 16.2\text{ kN}$

$P = T(2\text{ m/s}) = (16{,}200\text{ N})(2.0\text{ m/s}) = 32{,}400\text{ W} = 32\text{ kW}$

9.70. Model: Assume the spring is ideal so that Hooke's law is obeyed, and model the weather rocket as a particle.
Visualize:



The origin of the coordinate system is placed on the free end of the spring. Note that the bottom of the spring is anchored to the ground.

Solve: (a) The rocket is initially at rest. The free-body diagram on the rocket helps us write Newton's second law as

$$(\sum F_y) = 0 \text{ N} \;\Rightarrow\; F_{sp} = F_G = mg \;\Rightarrow\; k\Delta y = mg$$

$$\Delta y = \frac{mg}{k} = \frac{(10.2 \text{ kg})(9.81 \text{ m/s}^2)}{(500 \text{ N/m})} = 20.0 \text{ cm}$$

(b) The thrust does work. Using the energy conservation equation when $y_2 - y_e = 40 \text{ cm} = 0.40 \text{ m}$:

$$K_2 + U_{g2} + U_{sp2} = K_1 + U_{g1} + U_{sp1} + W_{ext}$$

$$W_{ext} = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}k(y_2 - y_e)^2 = \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(y_1 - y_e)^2 + (200 \text{ N})(0.60 \text{ m})$$

$$(5.10 \text{ kg})v_2^2 + 40.0 \text{ J} + 40.0 \text{ J} = 0 - 20.0 \text{ J} + 10.0 \text{ J} + 120 \text{ J} \;\Rightarrow\; v_2 = 2.43 \text{ m/s}$$

If the rocket were not attached to the spring, the energy conservation equation would not involve the spring energy term U_{sp2}. That is,

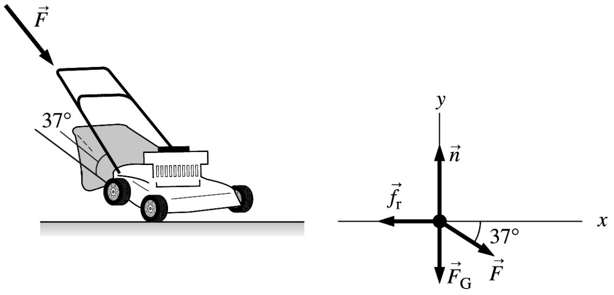
$$K_2 + U_{g2} = K_1 + U_{g1} + U_{sp1} + W_{ext}$$

$$\tfrac{1}{2}(10.2 \text{ kg})v_2^2 + (10.2 \text{ kg})(9.81 \text{ m/s}^2)(0.40 \text{ m}) = 0 \text{ J} - (10.2 \text{ kg})(9.81 \text{ m/s}^2)(0.20 \text{ m})$$
$$+ \tfrac{1}{2}(500 \text{ N/m})(0.20 \text{ m})^2 + (200 \text{ N})(0.60 \text{ m})$$

$$(5.10 \text{ kg})v_2^2 = 70.0 \text{ J} \;\Rightarrow\; v_2 = 3.70 \text{ m/s}$$

Assess: (a) The rocket has greater speed at y_2 when it is not attached to the spring because, as the spring extends, it contributes a downward force to the rocket.

9.71. Model: Model the lawnmower as a particle and use the model of kinetic friction.
Visualize:



We placed the origin of our coordinate system on the lawnmower and drew the free-body diagram of forces.
Solve: The normal force $\vec{n}$, which is related to the frictional force, is not equal to $\vec{F}_G$. This is due to the presence of $\vec{F}$. The rolling friction is $f_r = \mu_r n$, or $n = f_r/\mu_r$. The lawnmower moves at constant velocity, so $\vec{F}_{net} = \vec{0}$. The two components of Newton's second law are

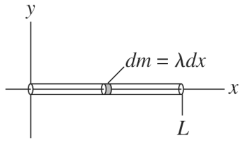
$$(\Sigma F_y) = n - F_G - F\sin(37°) = ma_y = 0\text{ N} \Rightarrow f_r/\mu_r - mg - F\sin(37°) = 0\text{ N} \quad \Rightarrow \quad f_r = \mu_r mg + \mu_r F\sin 37°$$

$$(\Sigma F_x) = F\cos(37°) - f_r = 0\text{ N} \quad \Rightarrow \quad F\cos(37°) - \mu_r mg - \mu_r F\sin(37°) = 0\text{ N}$$

$$F = \frac{\mu_r mg}{\cos(37°) - \mu_r\sin(37°)} = \frac{(0.15)(12\text{ kg})(9.8\text{ m/s}^2)}{0.7986 - (0.15)(0.6018)} = 24.9\text{ N}$$

Thus, the power supplied by the gardener in pushing the lawnmower at a constant speed of 1.2 m/s is $P = \vec{F}\cdot\vec{v} = Fv\cos\theta = (24.9\text{ N})(1.2\text{ m/s})\cos(37°) = 24\text{ W}$.

9.72. Model: Model the bar as one-dimensional.
Visualize: We must integrate the kinetic energy of each little element of the bar to get the total kinetic energy.



Known
$m = m$
$L = L$
$\omega = \omega$

Solve: In terms of the variables given:

$$K = \int dK = \int \tfrac{1}{2}v^2 dm = \tfrac{1}{2}\int(\omega x)^2 \lambda dx = \frac{1}{2}\frac{m}{L}\omega^2\left[\frac{1}{3}x^2\right]_0^L = \frac{1}{6}mL^2\omega^2$$

Assess: If we call $I = \frac{1}{3}mL^2$ for a bar rotated around one end, then $K = \frac{1}{2}I\omega^2$ as we expect by analogy with the linear case.